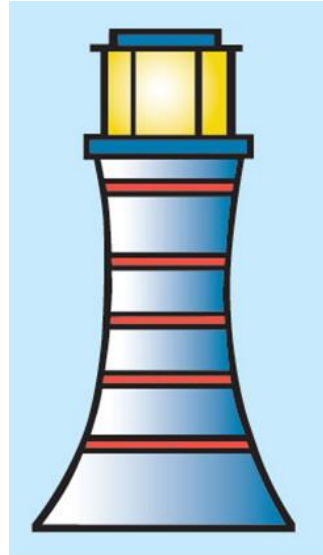


Austhorpe Primary School



CALCULATION POLICY

March 2021

Review date: March 2024

Introduction

The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school.

Please note that early learning in number and calculation in Reception follows the “Development Matters” EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

It is essential that when teaching calculation the three aims of the new National Curriculum 2014 are fulfilled – **Fluency, Problem Solving and Reasoning**.

Concrete, Pictorial and Abstract

Each calculation requirement in mathematics is broken down into **Concrete, Pictorial** and **Abstract** phases.

All children should be moving towards understanding and using all three forms of calculation and the policy does not directly attribute any specific skills to particular year group.

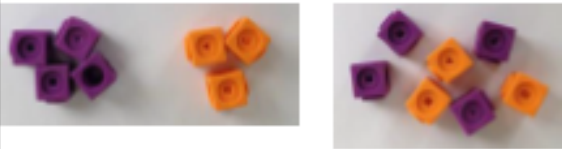
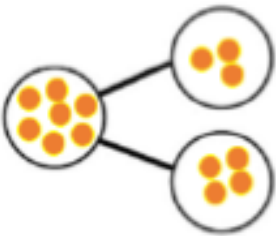
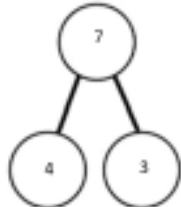
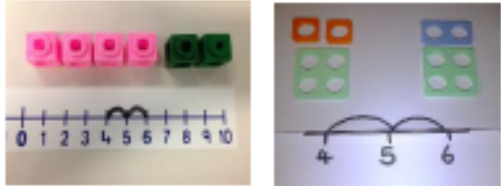
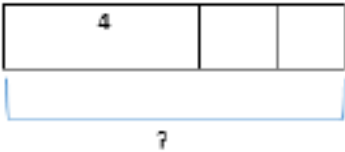
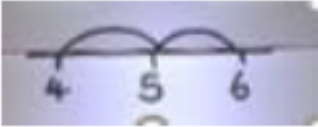

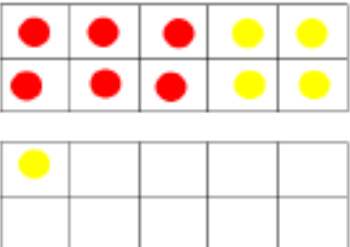
Concrete: using concrete resources to demonstrate mathematical applications to solve calculations

Pictorial: using images to represent mathematical applications to solve calculations

Abstract: using recognised mathematical methods to solve calculations

Addition

Key language which should be used: *sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' 'is the same as'*

Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears etc)</p> 	<p>Pictorial</p> 	<p>Abstract</p> <p>$4 + 3 = 7$ (four is a part, 3 is a part and the whole is seven)</p> 
<p>Counting on using number lines by using cubes or numicon</p> 	<p>A bar model which encourages the children to count on</p> 	<p>The abstract number line:</p> <p>What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2?</p> <p>$4 + 2$</p> 
<p>Regrouping to make 10 by using ten frames and counters/cubes or using numicon:</p> <p>$6 + 5$</p> 	<p>Children to draw the ten frame and counters/cubes</p> 	<p>Children to develop an understanding of equality e.g $6 + \square = 11$ and</p> <p>$6 + 5 = 5 + \square$ $6 + 5 = \square + 4$</p>

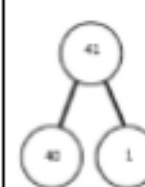
TO + O using base 10. Continue to develop understanding of partitioning and place value
 $41 + 8$



Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones.



$41 + 8$



$$1 + 8 = 9$$

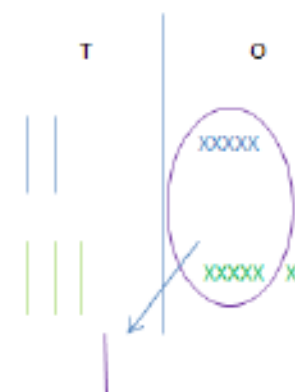
$$40 + 9 = 49$$

	4	1
+		8
<hr/>		
	4	9

TO + TO using base 10. Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging. $36 + 25$

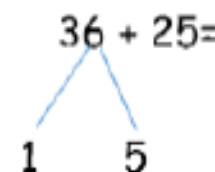
	Tens	Ones
+		
=		

This could be done one of two ways:



Tens	Ones

Looking for ways to make 10



$$30 + 20 = 50$$

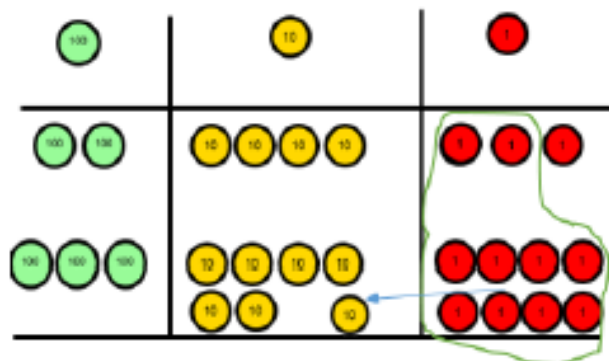
$$5 + 5 = 10$$

$$50 + 10 + 1 = 61$$

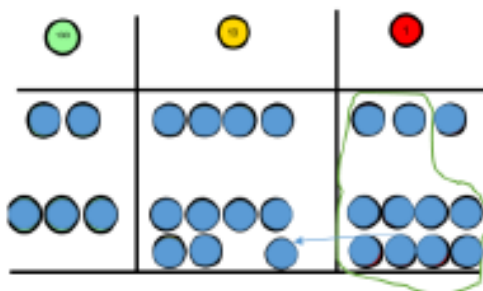
Formal method:

$$\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ \hline 1 \end{array}$$

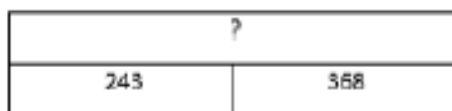
Use of place value counters to add HTO + TO, HTO + HTO etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract



Children to represent the counters e.g. like the image below



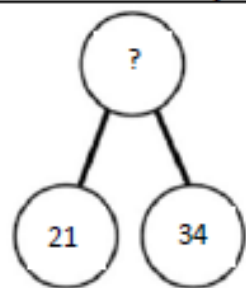
If the children are completing a word problem, draw a bar model to represent what it's asking them to do



243

$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ 1 \quad 1 \end{array}$$

Fluency variation, different ways to ask children to solve 21+34:



Sam saved £21 one week and £34 another. How much did he save in total?

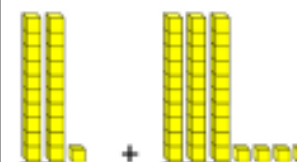
$21+34=55$. Prove it! (reasoning but the children need to be fluent in representing this)

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$$21 + 34 =$$

$$\square = 21 + 34$$

What's the sum of twenty one and thirty four?





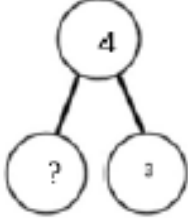
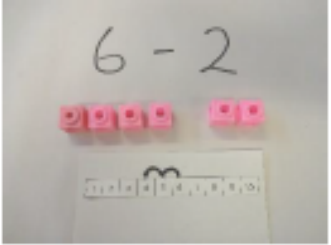
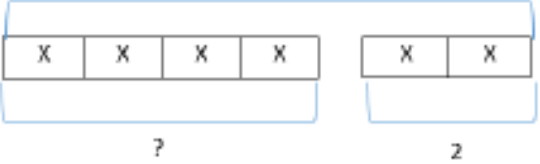
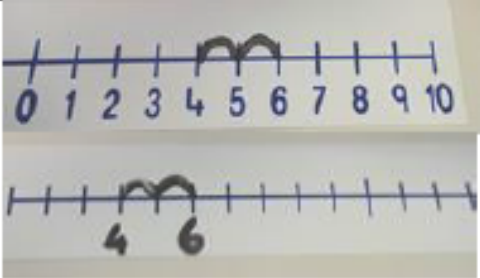


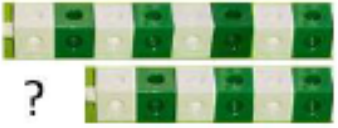
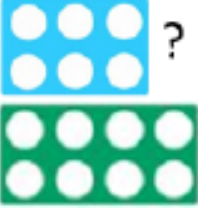

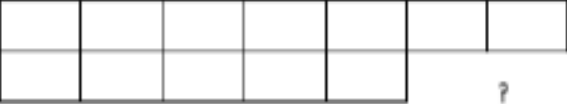


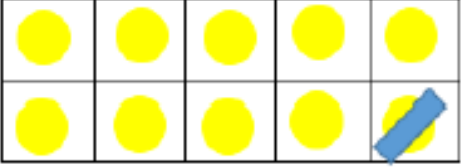

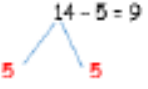


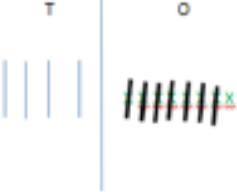
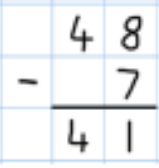
Always use missing digit problems too:


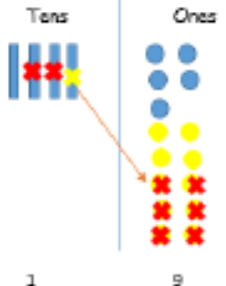

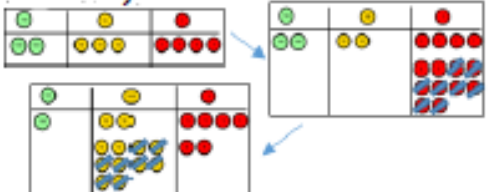
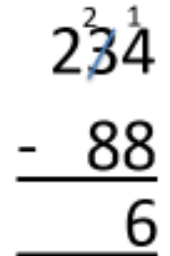
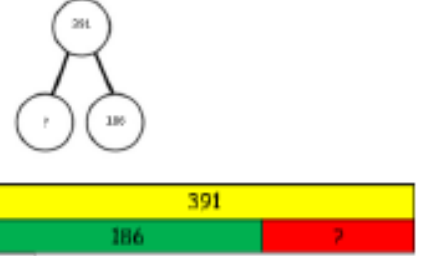
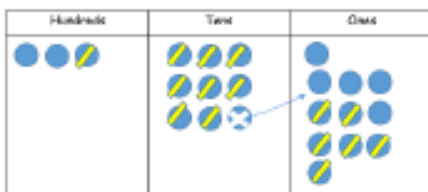
Tens	Ones
● ●	●
● ● ●	?
?	4

Subtraction

Key language which should be used: take away, less than, the difference, subtract, minus, fewer, decrease, '7 take away 3, the difference is four'



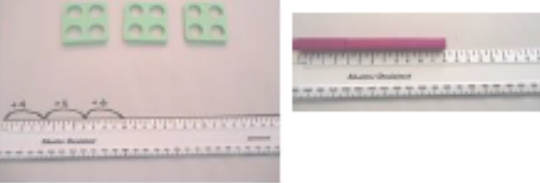
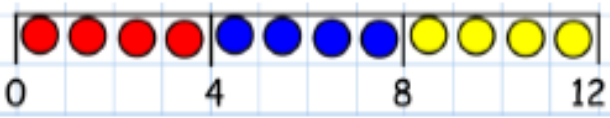
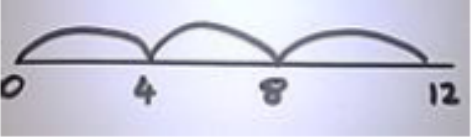

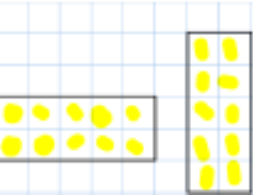
Concrete	Pictorial	Abstract
<p>Physically taking away and removing objects from a whole (use various objects too) rather than crossing out- children will physically remove the objects</p> <p>$4 - 3 = 1$</p> 	<p>Children to draw the concrete resources they are using and cross out.</p>  <p>Use of the bar model:</p> 	<p>$4 - 3 =$</p> <p><input type="text"/> = $4 - 3$</p>  
<p>Counting back (using number lines or number tracks)</p> 	<p>Children to represent what they see pictorially e.g.</p> 	

<p>Finding the difference (using cubes, numicon or Cuisenaire rods, other objects can also be used)</p>   	<p>Children to draw the cubes/other concrete objects which they have used</p> <p>XXXXXXXXXX XXXXXXX</p> <p>Use of the bar model</p> 	<p>Find the difference between 8 and 6.</p> <p>$8 - 6$, the difference is ?</p> <p>Children to also explore why $9 - 7 = 8 - 6$ (the difference, of each digit, has changed by 1 do the difference is the same- this will help when solving 10000-9987)</p>
<p>Making 10 (using numicon or ten frames)</p> <p>$14 - 5$</p>  <p>Children could also do this by subtracting a 5 from the 10.</p> 	<p>Children to present the ten frame pictorially</p>  	<p>$14 - 5 = 9$ You also want children to see related facts e.g. $15 - 9 = 5$</p> <p>Children to represent how they have solved it e.g.</p>  <p>$14 - 5 = 9$ 14 is made up of 5, 5 and 4 so I can subtract one 5 to be left with 4 and 5</p>  <p>$14 - 5 = 9$ 5 is made up of 4 and 1 so I can subtract 4 to make 10 and then 1 to get to 9</p>
<p>Column method (using base 10)</p> <p>$48 - 7$</p> 		<p>$48 - 7 =$</p> 

<p>Column method (using base 10 and having to exchange) 45-26</p>  <ol style="list-style-type: none"> 1) Start by partitioning 45 2) Exchange one ten for ten more ones 3) Subtract the ones, then the tens. 	<p>Represent the base 10 pictorially</p> 	<p>It's crucial that the children understand that when they have exchanged the 10 they still have 45. $45 = 30 + 15$</p> 	
<p>Column method (using place value counters) 234-88</p> 	<p>Once the children have had practice with the concrete, they should be able to apply it to any subtraction.</p> <p>Like the other pictorial representations, children to represent the counters.</p>		
<p>Fluency variation, different ways to ask children to solve 391-186:</p>			
	<p>Raj spent £391, Timmy spent £186. How much more did Raj spend?</p> <p>I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?</p>	<p>$391 - 186$</p> <p><input type="text"/> = $391 - 186$</p> $\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$ <p>Find the difference between 391 and 186 Subtract 186 from 391. What is 186 less than 391?</p>	<p>What's the calculation? What's the answer?</p>  $\begin{array}{r} 39\text{ } \\ - \text{ } \text{ } 6 \\ \hline \text{ } 0 \text{ } 5 \end{array}$

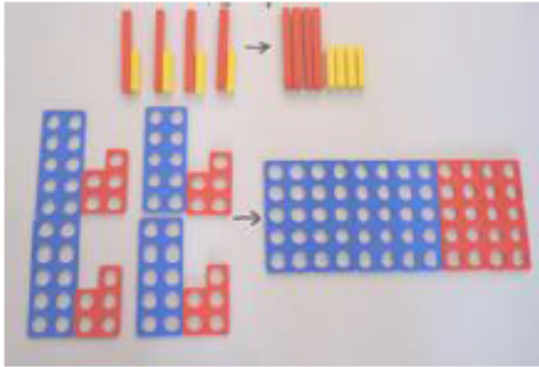
Multiplication

Key language which should be used: double times, multiplied by, the product of, groups of, lots of, 'is equal to' 'is the same as'

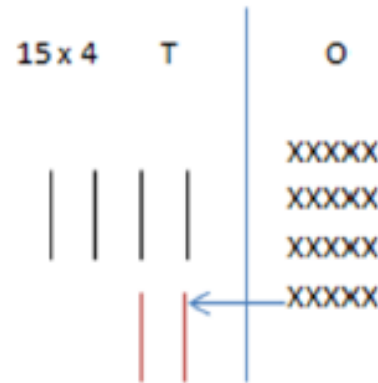
Concrete	Pictorial	Abstract
<p>Repeated grouping/repeated addition (does not have to be restricted to cubes) 3×4 or 3 lots of 4</p> 	<p>Children to represent the practical resources in a picture e.g.</p> <p>XX XX XX XX XX XX</p> <p>Use of a bar model for a more structured method</p> 	<p>3×4</p> <p>$4 + 4 + 4$</p>
<p>Use number lines to show repeated groups- 3×4</p> 	<p>Represent this pictorially alongside a number line e.g:</p> 	<p>Abstract number line $3 \times 4 = 12$</p> 
<p>Use arrays to illustrate commutativity (counters and other objects can also be used) $2 \times 5 = 5 \times 2$</p> 	<p>Children to draw the arrays</p> 	<p>Children to be able to use an array to write a range of calculations e.g.</p> <p>$2 \times 5 = 10$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $5 + 5 = 10$</p>

Partition to multiply (use numicon, base 10, Cuisenaire rods)

4×15



Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like:

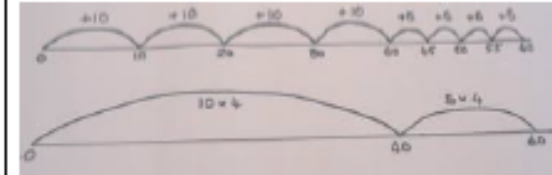


Children to be encouraged to show the steps they have taken

$$\begin{array}{r} 4 \times 15 \\ \swarrow \searrow \\ 10 \quad 5 \end{array}$$

$$\begin{array}{l} 10 \times 4 = 40 \\ 5 \times 4 = 20 \\ 40 + 20 = 60 \end{array}$$

A number line can also be used

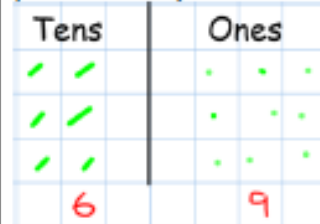


Formal column method with place value counters or base 10 (at the first stage- no exchanging) 3×23

Make 23, 3 times. See how many ones, then how many tens



Children to represent the counters in a pictorial way



Children to record what it is they are doing to show understanding

$$\begin{array}{r} 3 \times 23 \\ \swarrow \searrow \\ 20 \quad 3 \end{array} \quad \begin{array}{l} 3 \times 20 = 60 \\ 3 \times 3 = 9 \\ 60 + 9 = 69 \end{array}$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

Formal column method with place value counters (children need this stage, initially, to understand how the column method works)

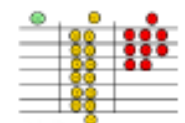
Children to represent the counters/base 10, pictorially e.g. the image below.

$$\begin{array}{l} 6 \times 23 \\ 6 \times 3 = 18 \\ 6 \times 20 = 120 \\ 120 + 18 = 138 \end{array}$$

6×23



Step 1: get 6 lots of 23



Step 2: 6×3 is 18. Can I make an exchange? Yes! Ten ones for one ten....



Step 3: 6×2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred....



Step 4- what do I have I each column?



The aim is to get to the formal method but the children need to understand how it works.

$$\begin{array}{r}
 6 \times 23 = \\
 23 \\
 \times 6 \\
 \hline
 138 \\
 \hline
 1 \quad 1
 \end{array}$$

When children start to multiply $3d \times 3d$ and $4d \times 2d$ etc, they should be confident with the abstract:

To get 744 children have solved 6×124
 To get 2480 they have solved 20×124

$$\begin{array}{r}
 1 \quad 2 \quad 4 \\
 \times \quad 2 \quad 6 \\
 \hline
 7 \quad 4 \quad 4 \\
 \\
 2 \quad 4 \quad 8 \quad 0 \\
 \hline
 3 \quad 2 \quad 2 \quad 4 \\
 \hline
 1 \quad 1
 \end{array}$$

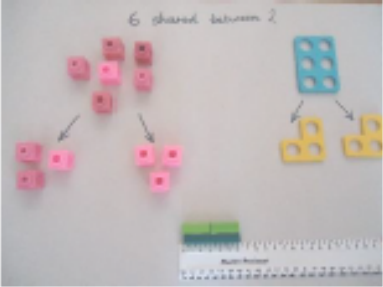


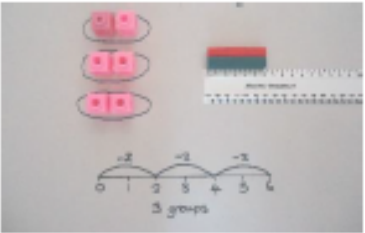
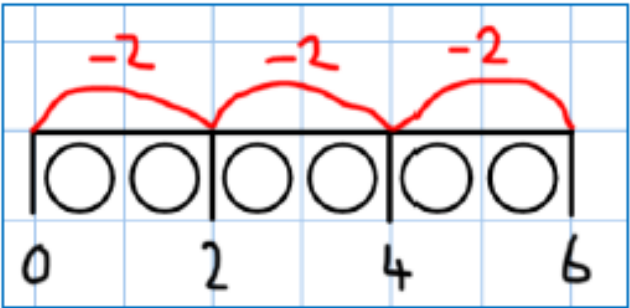
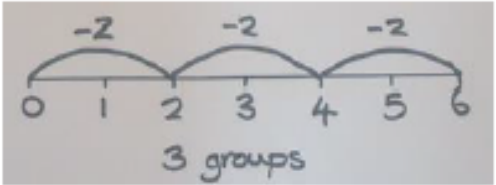
Answer: 3224


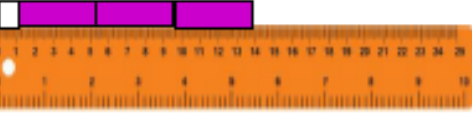
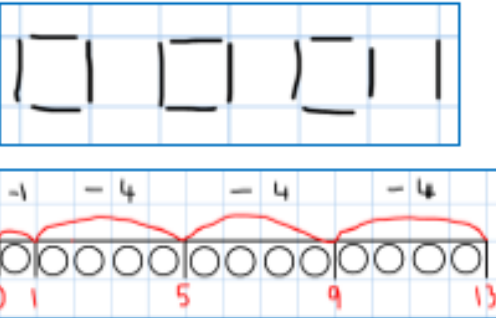
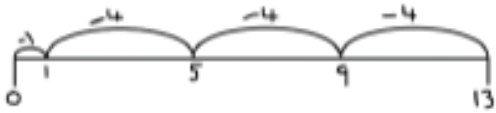
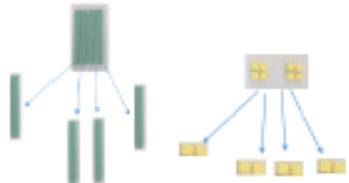
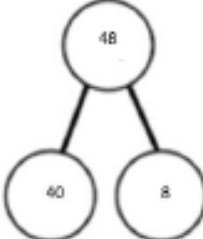

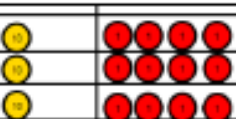
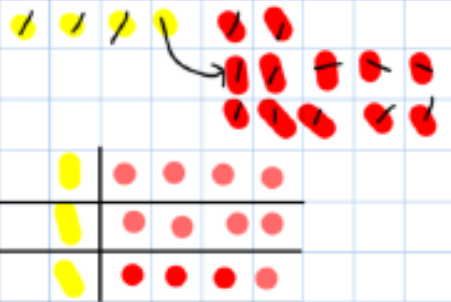
Fluency variation, different ways to ask children to solve 6×23 :

<div data-bbox="129 228 533 316"> <table border="1"> <tr> <td>23</td> <td>23</td> <td>23</td> <td>23</td> <td>23</td> <td>23</td> </tr> </table> </div> <div data-bbox="331 331 353 363">?</div> <p data-bbox="118 419 548 491">With the counters, prove that $6 \times 23 = 138$</p> <p data-bbox="118 539 436 571">Why is $6 \times 23 = 32 \times 6$?</p>	23	23	23	23	23	23	<p data-bbox="589 180 929 331">Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?</p> <p data-bbox="589 379 929 491">Tom saved 23p three days a week. How much did he save in 2 weeks?</p>	<p data-bbox="969 180 1310 252">Find the product of 6 and 23</p> <p data-bbox="969 300 1086 331">$6 \times 23 =$</p> <p data-bbox="969 371 1153 411"><input type="text"/> = 6×23</p> <div data-bbox="969 419 1176 531"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 10px;">6</td> <td style="padding-right: 10px;">23</td> <td></td> </tr> <tr> <td style="border-bottom: 1px solid black;">x 23</td> <td style="border-bottom: 1px solid black;">x 6</td> <td></td> </tr> <tr> <td style="border-bottom: 1px solid black;">—</td> <td style="border-bottom: 1px solid black;">—</td> <td></td> </tr> </table> </div>	6	23		x 23	x 6		—	—		<p data-bbox="1373 180 1848 252">What's the calculation? What's the answer?</p> <div data-bbox="1373 260 1758 507"> </div>
23	23	23	23	23	23													
6	23																	
x 23	x 6																	
—	—																	

Division

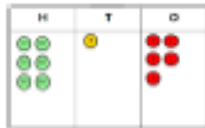
Key language which should be used: share, group, divide, divided by, half, 'is equal to' 'is the same as'

Concrete	Pictorial	Abstract		
<p>6 shared between 2 (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates)</p>  <p>A photograph showing 6 small pink objects being divided into two groups of three. A blue tray with 6 compartments and two yellow trays with 3 compartments each are also shown.</p>	<p>Pictorial</p>  <p>Two circles, each containing three small white dots, representing 6 objects shared between 2 groups.</p> <p>This can also be done in a bar so all 4 operations have a similar structure:</p>  <p>A bar divided into two equal sections, each containing three blue dots, representing 6 objects shared between 2 groups.</p>	<p>$6 \div 2 = 3$</p> <p>What's the calculation?</p> <table border="1" data-bbox="1384 421 1823 491"> <tr> <td>3</td> <td>3</td> </tr> </table>	3	3
3	3			
<p>Understand division as repeated grouping and subtracting $6 \div 2$</p>  <p>A photograph showing 6 small pink objects grouped into three pairs of two. A number line below shows 0 to 6 with three jumps of 2, labeled '3 groups'.</p>	 <p>A number line from 0 to 6 with circles at each number. Three red arcs above the line, each labeled '-2', show jumps from 0 to 2, 2 to 4, and 4 to 6.</p>	<p>Abstract number line</p>  <p>A number line from 0 to 6 with three arcs above it, each labeled '-2', showing jumps from 0 to 2, 2 to 4, and 4 to 6. Below the line, it says '3 groups'.</p>		
<p>2d ÷ 1d with remainders $13 \div 4 = 3$ remainder 1</p>	<p>Children to have chance to represent the resources they use in a pictorial way e.g. see below:</p>	<p>$13 \div 4 = 3$ remainder 1</p> <p>Children to count their times tables facts in their heads</p>		

<p>Use of lollipop sticks to form wholes</p>  <p>Use of Cuisenaire rods and rulers (using repeated subtraction)</p> 		
<p>2d divided by 1d using base 10 (no remainders) SHARING</p> <p>$48 \div 4 = 12$</p>  <p>Start with the tens.</p>	<p>Children to represent the base 10 and sharing pictorially.</p>	<p>$48 \div 4$</p> <p>4 tens $\div 4 = 1$ ten 8 ones $\div 4 = 2$ ones</p> <p>$10 + 2 = 12$</p> 
<p>Sharing using place value counters.</p> <p>$42 \div 3 = 14$</p>  <p>1. Make 42. Share the 4 tens between 3. Can we make an exchange with the extra 10?</p>  <p>Exchange the ten for 10 ones and share out 12 ones</p>		<p>$42 \div 3$</p> <p>$42 = 30 + 12$</p> <p>$30 \div 3 = 10$</p> <p>$12 \div 3 = 4$</p> <p>$10 + 4 = 14$</p>

Use of the 'bus stop method' using grouping and counters. Key language for grouping- how many groups of X can we make with X hundreds'- *this can also be done using sharing!*

$$615 \div 5$$



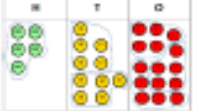
Step 1: make 615



Step 2: Circle your groups of 5



Step 3: Exchange 1H for 10T and circle groups of 5



Step 4: exchange 1T for 10ones and circles groups of 5

This can easily be represented pictorially, till the children no longer to do it. It can also be done to decimal places if you have a remainder!

$$5 \overline{) 615}$$

Fluency variation, different ways to ask children to solve $615 \div 5$:

Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

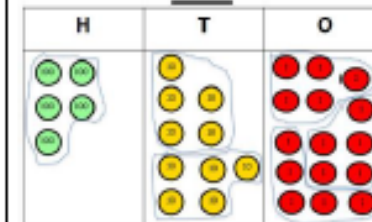
$$5 \overline{) 615}$$

$$615 \div 5 =$$




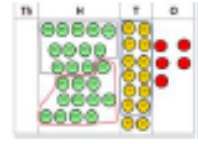
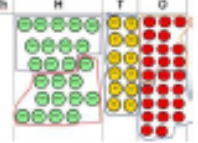
$$\square = 615 \div 5$$

How many 5's go into 615?

What's the calculation? What's the answer?



Long Division

Concrete	Pictorial	Abstract
 $\begin{array}{r} 0212 \\ 12 \overline{)2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$ <p>2544 ÷ 12 How many groups of 12 thousands do we have? None</p>	<p>Children to represent the counters, pictorially and record the subtractions beneath.</p>	$12 \overline{)2544}^0$ <p>Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.</p>
 <p>Exchange 2 thousand for 20 hundreds.</p>		$12 \overline{)2544}^{02}$ <p>Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.</p>
 $12 \overline{)2544}^{02}$ <p>How many groups of 12 are in 25 hundreds? 2 groups. Circle them.</p>		$12 \overline{)2544}^{021}$ <p>Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens?</p>
<p>We have grouped 24 hundreds so can take them off and we are left with one.</p>		<p>The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.</p>
 $12 \overline{)2544}^{021}$ <p>Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.</p>		$12 \overline{)2544}^{0212}$ <p>Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.</p>
 <p>Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2</p>		

